Change-point prediction/detection as alternative to insurer traditional hedging for equity benefit guarantees

EcoSta 2026 Session EV274: virtual Room 209

Multivariate and high dimensional methods

Sunday 8/9/2029

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https://ihaconsultants.com

Outline:

1.) AI-enabled Predictive Equity Analytics:

Model
Signals
Concepts
Changepoints-retrospective and prospective

2.) Distribution Ink Blots as informative leading to Discrete Fokker-Planck

3.)Current Hedging Life / Annuity Products

4.) AI-enabled Predictive Equity Analytics Hedging Life / Annuity Products

5.) Summary

Download zip file of Excel predictive reports available at:
https://ihaconsultants.com/trisignal_sample_report/sample_trisignalpea_predictive_reports_etfs_mutual_funds_stocks_2026_08_03.zip

AI-enabled Predictive Equity Analytics:

Model

Signals

Concepts

Changepoints-retrospective and prospective

AI-enabled Predictive Equity Analytics foundational model

Predictive Equity Analytics with Dynamic State Space
Definitions:
Position,
Momentum

$$r_t = \mu_t + f(\sigma_t^2) + I\_jump\_process_t$$

Where return (r_t) is a function of drift (μ_t), volatility (σ_t) and jumps ($I\_jump\_process$)

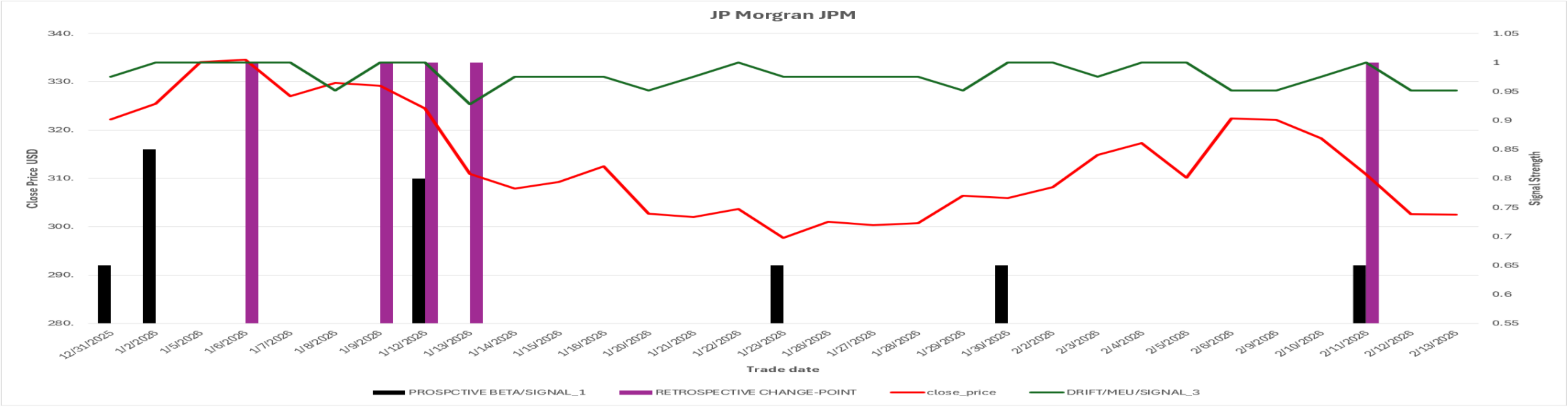
- Predictive Equity Analytics with Dynamic State Space **position**
  – defined as the ***level of*** μ_t
- Predictive Equity Analytics with Dynamic State Space **momentum**
  – defined as the ***rate of change of*** μ_t

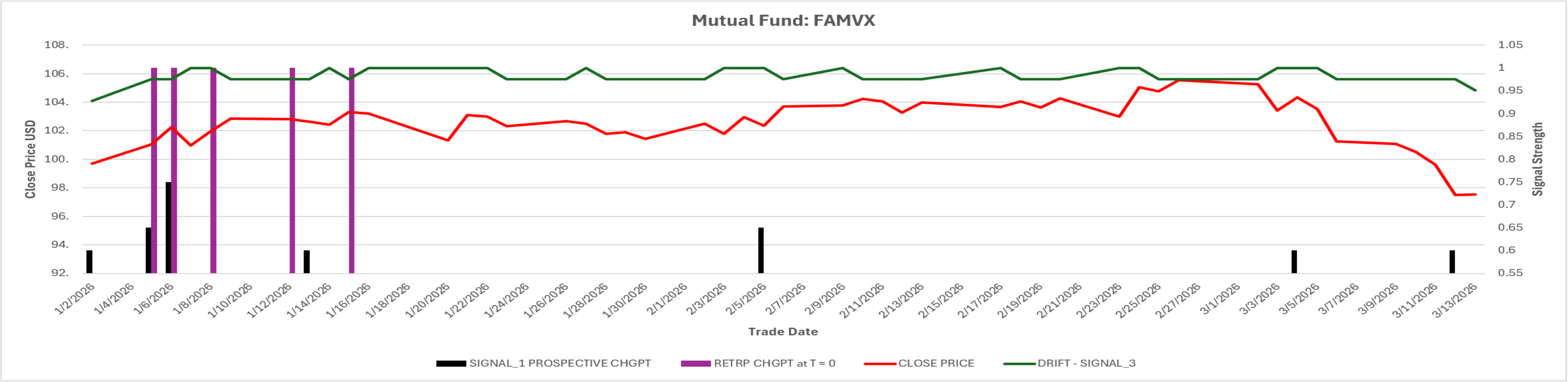
Cf. the **standard momentum** considered in the literature:

- Time series momentum
  – **A security's *own* past return predicts its future return**
- Cross-sectional momentum
  – **A security's *outperformance relative to peers* predicts future *relative outperformance***

Moskowitz, Tobias J., Yao Hua Ooi, and Lasse Heje Pedersen. 2012. "Time Series Momentum." Journal of Financial Economics 104 (2): 228–250. doi:10.1016/j.jfineco.2011.11.003.

Prospective Signal_1-Drift, Prospective Signal_3-local variance and Retrospective change-points are informative signals for any equity time-series.



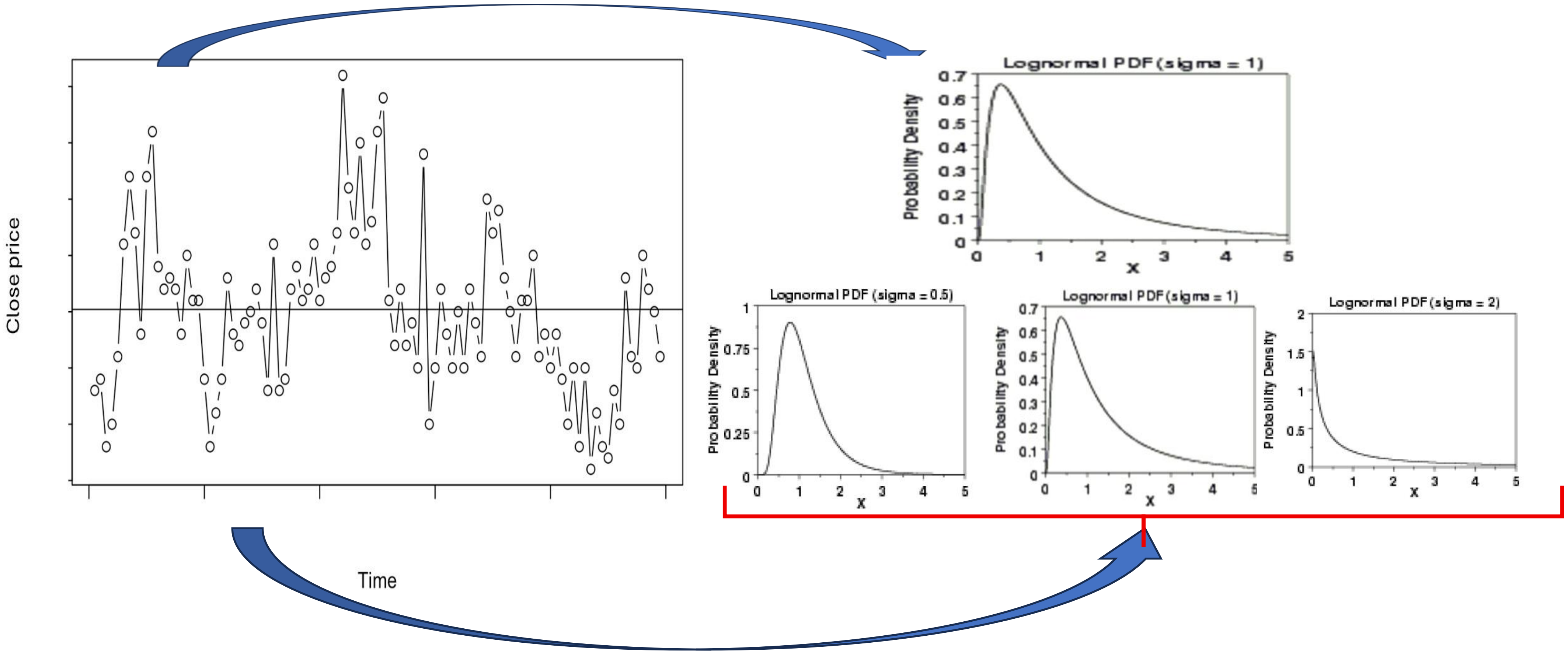


AI-enabled Predictive Equity Analytics (PEA) defined

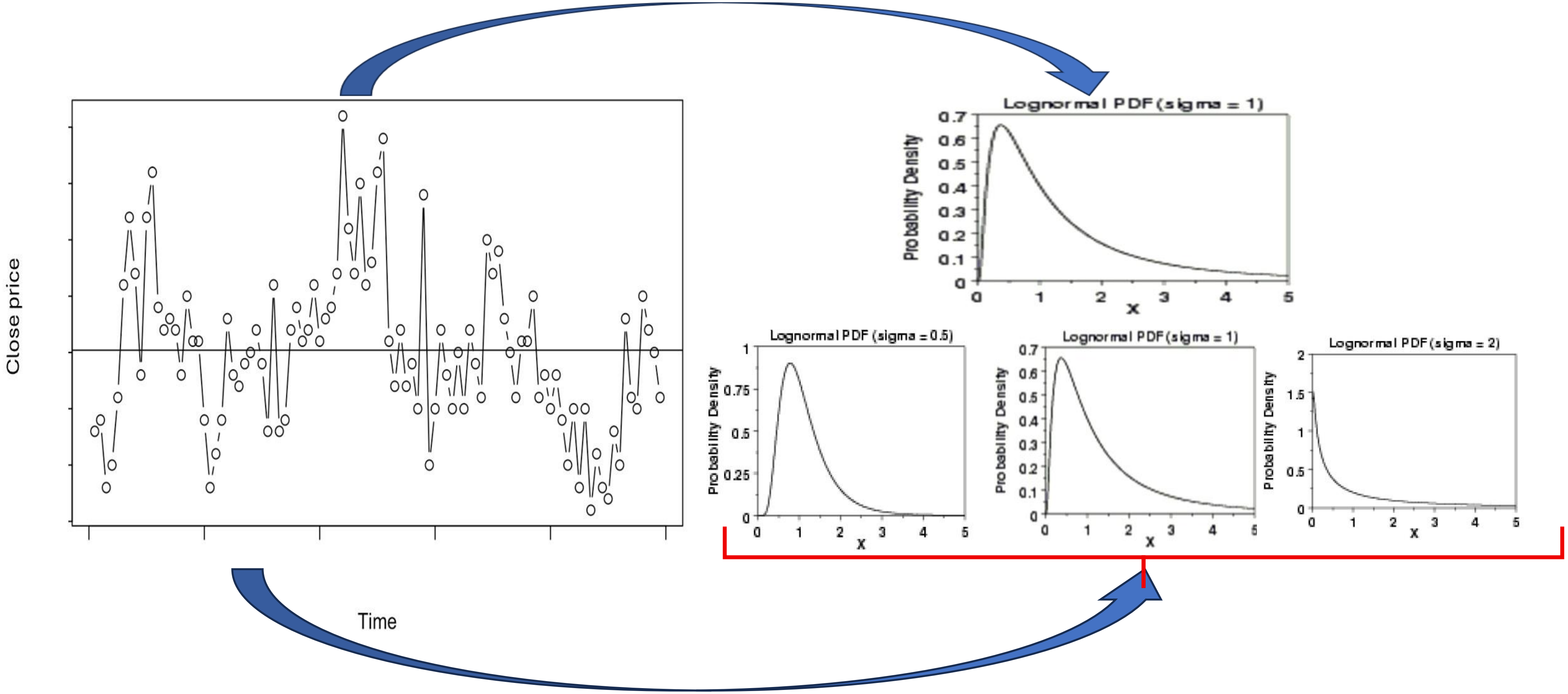
Simply, it is the capability to retrospectively as well as prospectively predict/detection change points and associated subsequent time-series behavior for ETFs, mutual funds and stocks with high accuracy.
This is commonly referred to as alpha signal prediction/detection in as little as 1 elapsed trade day.
When significant signals are present, predictive AI-enabled equity analytics enables greater investment returns than Markowitz based portfolios at significantly less risk levels.
The presence of significant signals means that the equity time series derives from a dynamic mixture of distributions rather than a single common distribution.
In absence of significant signals, predictive AI-enabled equity analytics gracefully reverts to standard theory.
Predictive AI-enabled equity analytics extends standard theory to accommodate non-equilibrium dynamics as well as dynamic mixture of distributions.
Predictive AI-enabled equity analytics’ support is derived from two sources: a) statistically significant T-tests and ANOVA tests and b) linear algebra: principal component method – maximal eigenvalue and associated eigenvector.

Merton type stochastic return model → Signal set extraction → Prospective/Retrospective Change-point Prediction/Detection

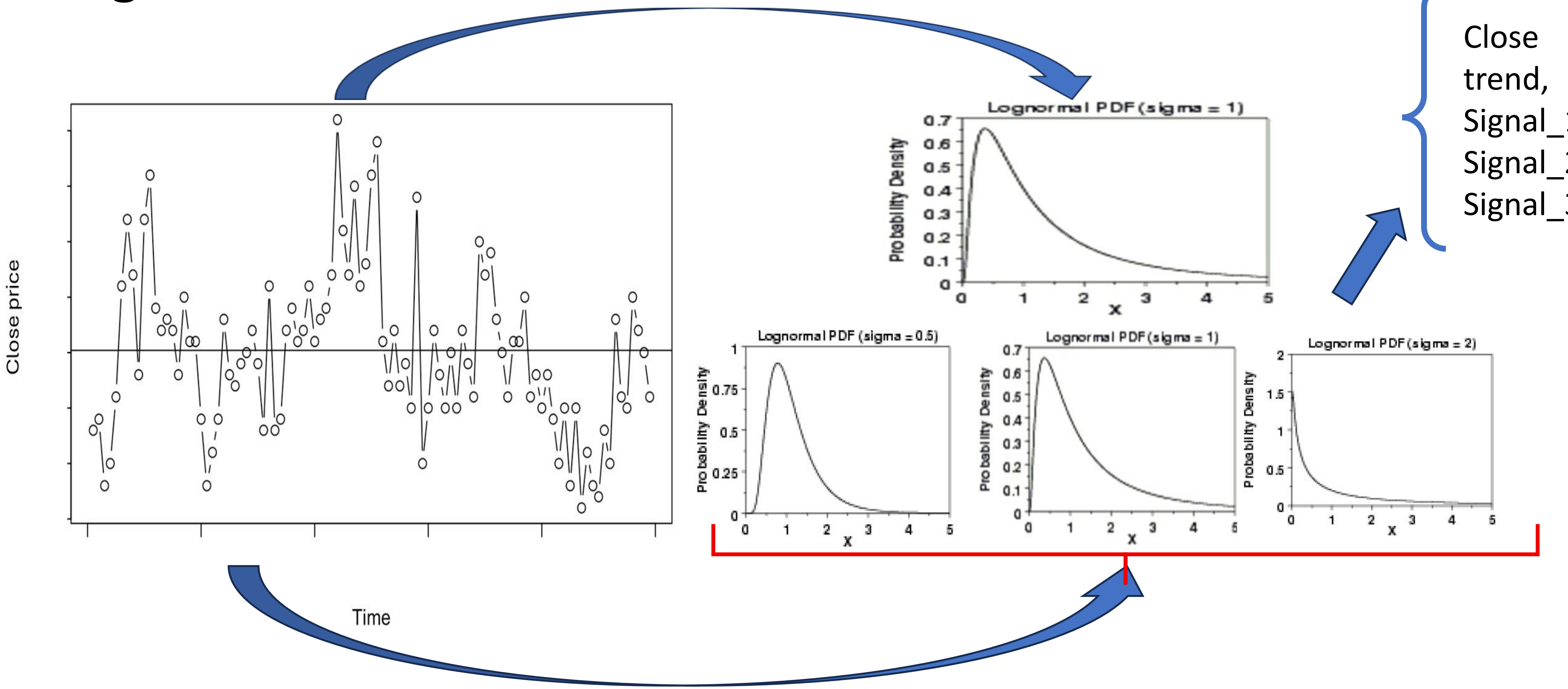
Predictive Equity Analytics Starting Concept 0: dynamic analysis is utilized to understand/visualize the morphing of the underlying statistical distributions or distribution mixture with respect to time and state space: a.k.a. discrete Fokker-Planck approximation.



Predictive Equity Analytics Starting Concept 1: is the underlying distribution supporting an equity time series a single common distribution or is a mixture of distributions a better representation.

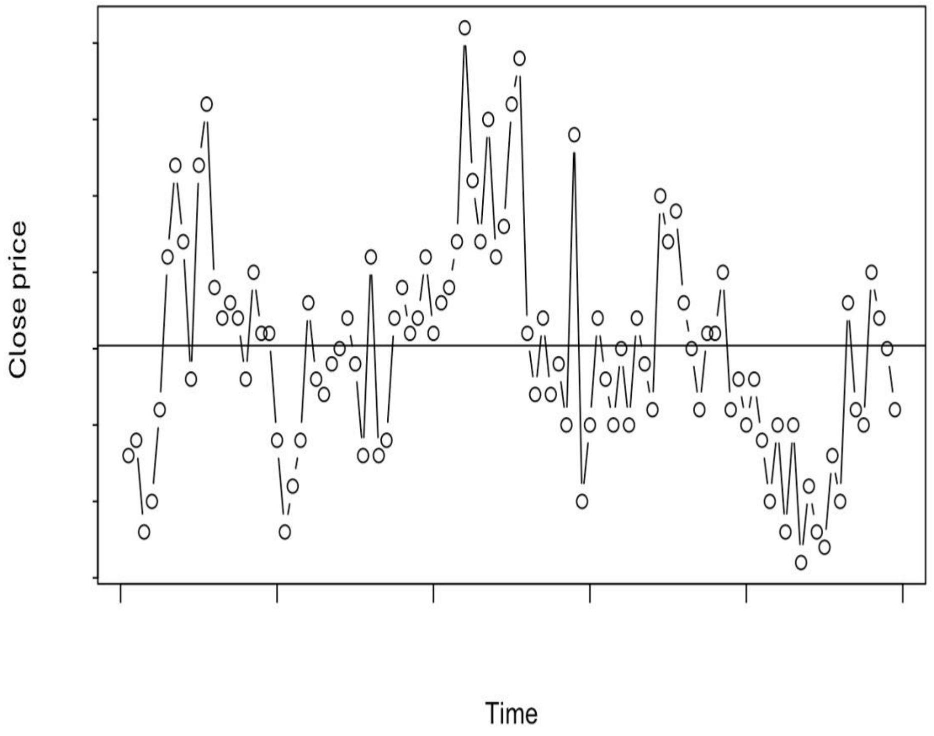


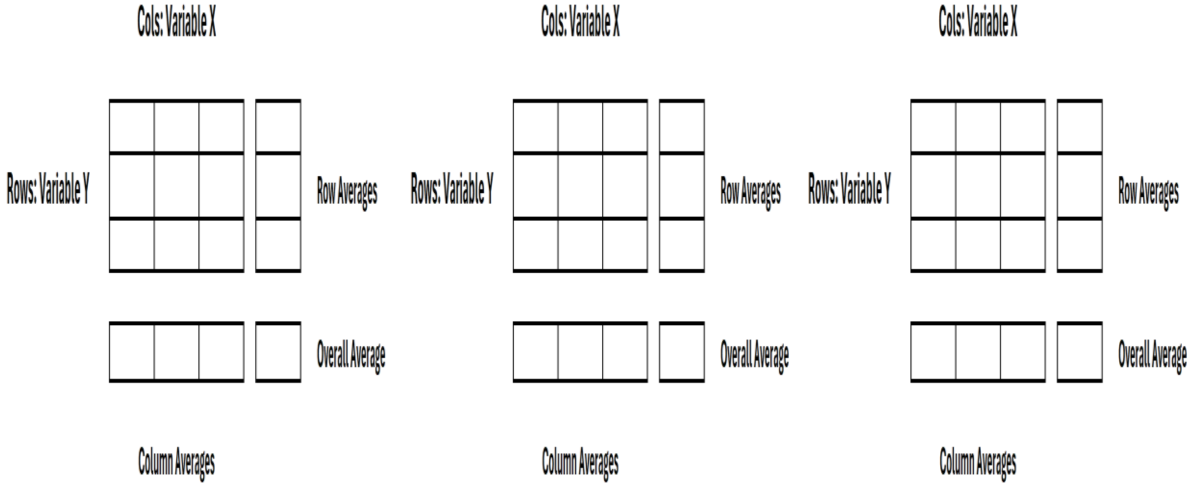
Predictive Equity Analytics Starting Concept 2: can informative internal signals be detected for each observed point that supports single common or distribution mixture?



Close trend, Signal_1, Signal_2, Signal_3,

Predictive Equity Analytics Starting Concept 3: internal signals be detected for each observed point then mapped to a set of 3 x 3 matrices with pair-wise T-testing row and column averages to test single distribution assumption.





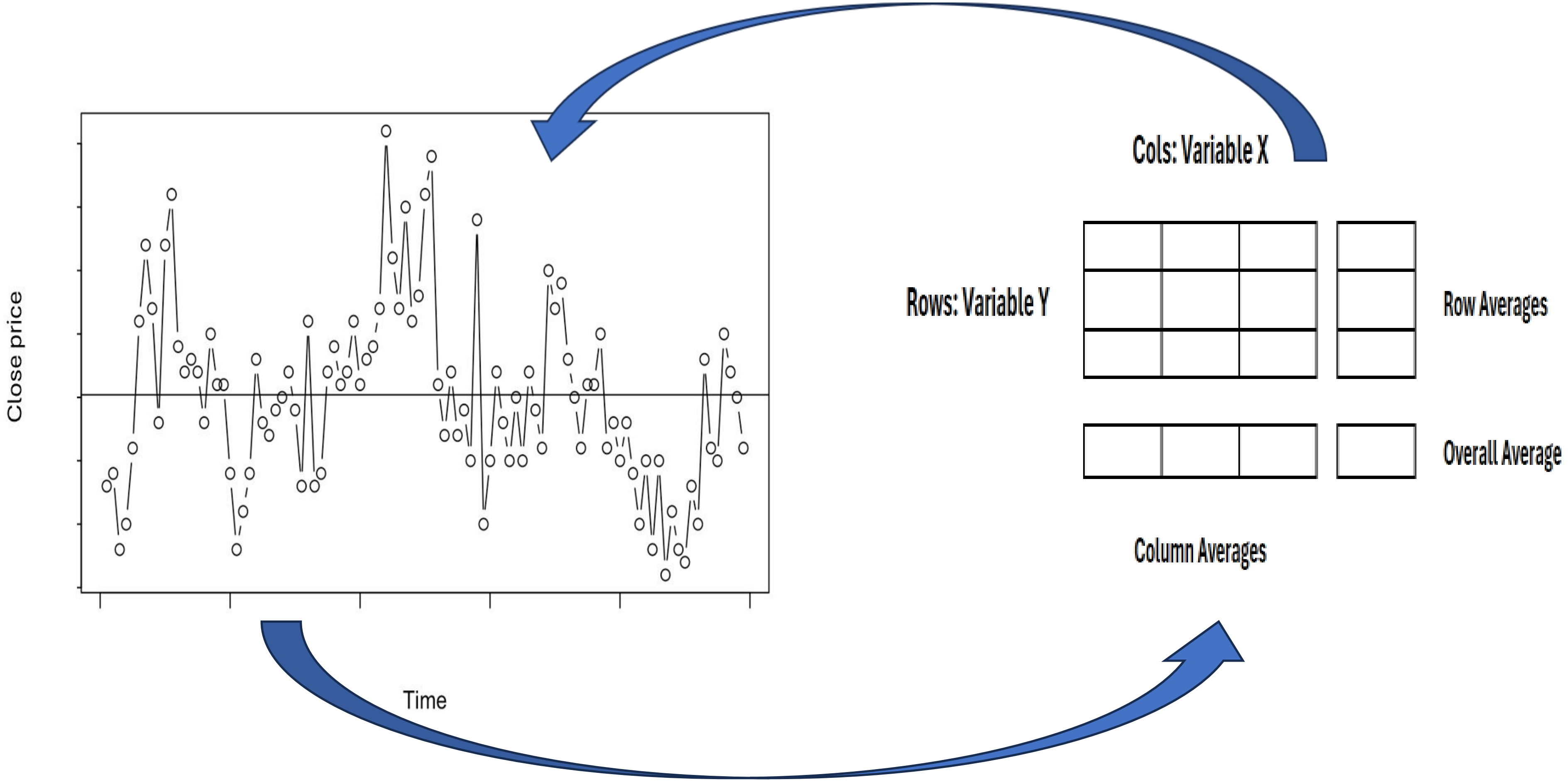
Matrix A
High variance
Poisson Point
Process

Matrix B
High variance

Matrix C
Filtered Signal

Raw signal = A+B+C → "ABC"
Variance = A+B
Filtered Signal = C

Predictive Equity Analytics Key: the mapping method employed to transform equity time series point-wise into a set of 3 x 3 matrices representation.



Desired Tests

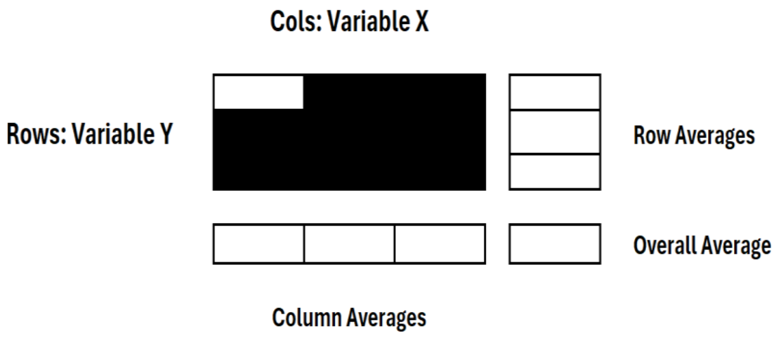
Statistical Tests:
1.) ANOVA
2.) Pair-wise T-tests for rows and columns

Linear Algebra: Principal Component Analysis/Eigen-analysis

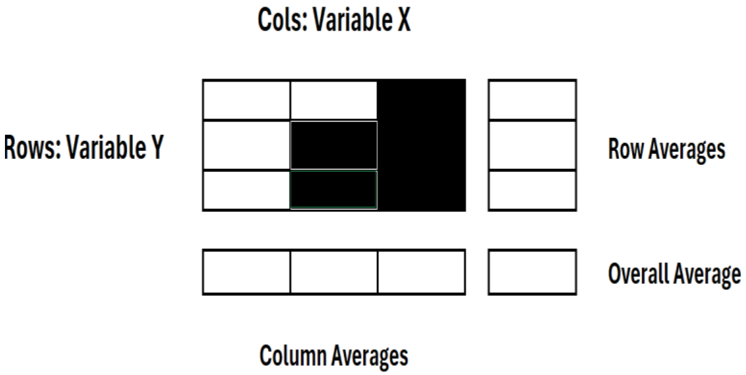
Distribution Ink Blots as informative leading to Discrete Fokker-Planck

Examples of the set of matrices {A,B,C} have consistent internal order. "Distribution ink blots"

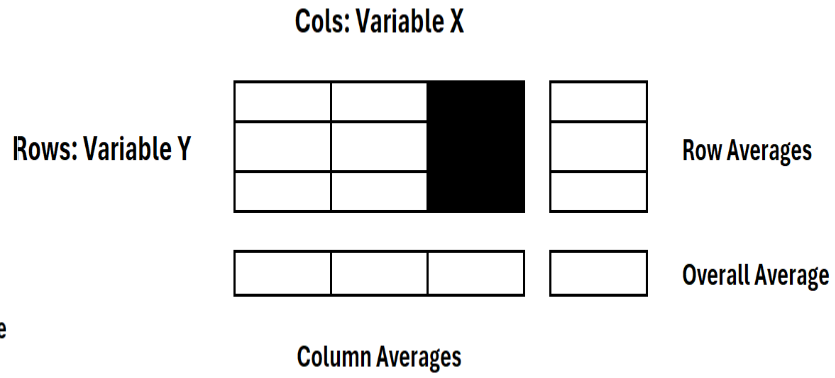
Stock



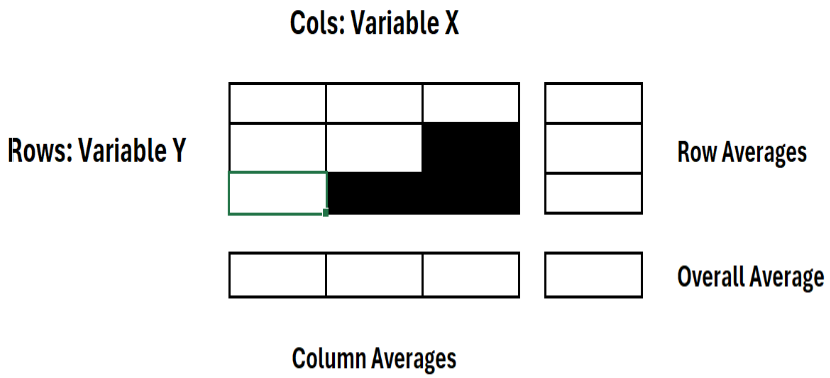
ETF



Mutual Fund



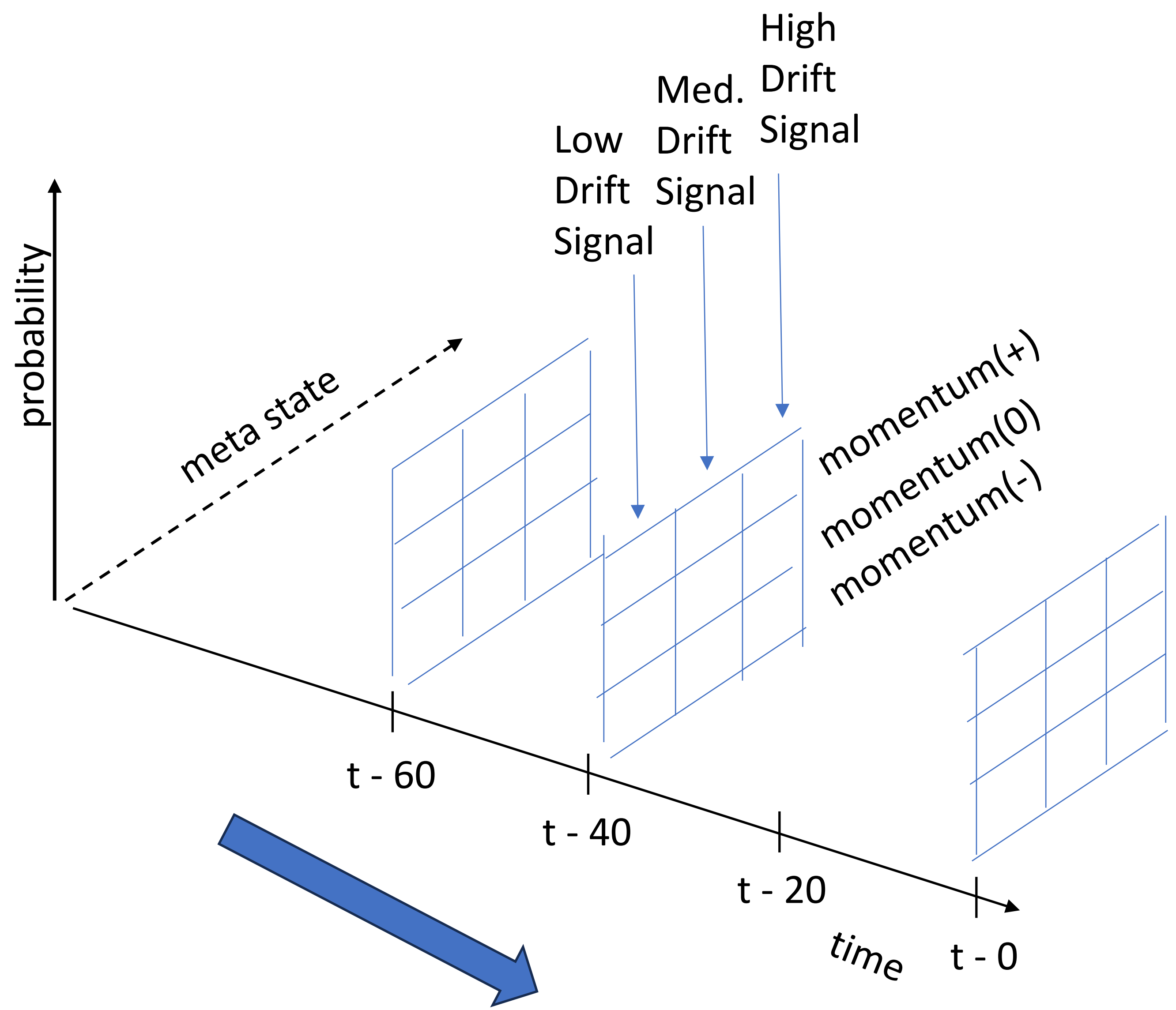
Mutual Fund



Point-wise mapping of equity time-series into set of 3x3 matrices analysis discussion.

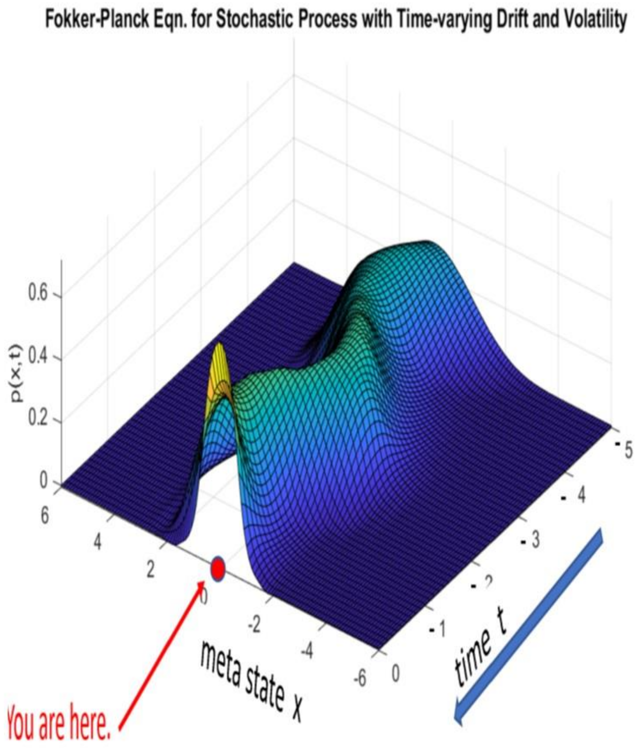
- **Case 1:** Mapping is **random** which results in random matrices or **inconsistent** sets of non-random matrices. Matrices fail row and column ANOVA and pair-wise T-Tests but do have principal eigenvalue and associated eigenvector. **This case belongs to the study of random matrices. Random matrix theory concerns detecting signals from combined signal plus noise data points in the aggregate. Signal intensity (eigen values and associated eigen vectors) must be sufficiently strong above a threshold to be detected against combined signal plus noise eigen spectrum.**
- **Case 2:** Mapping is **non-random** **(i.e. informative)** which results in **consistently** informative set of matrices which fail random matrix test. Matrices have either significant row and column ANOVA (or both) and 1 or more significant pair-wise (row/column) T-Tests as well as principal eigenvalue and associated eigenvector. **This case belongs to the new point-wise line of analysis called predictive equity analytics study – the study of prospective and retrospective change-points in equity time-series.**
- Note: Predictive equity analytics informative mapping is unique at each trade date measurement in the sense that we are observing non-random matrices.

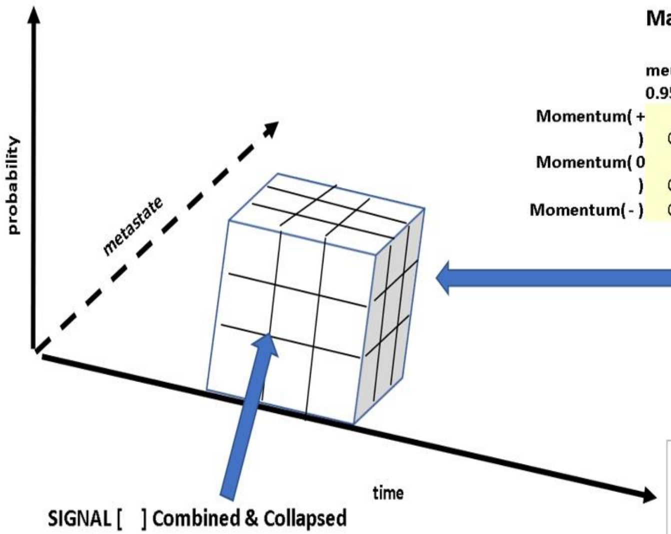
Matrix Theory Representation as Complete Alternative Measurement Framework – Changes over Time and over Meta State Matrices



PREVIEW:

Predictive Equity Analytics with Dynamic State Encapsulates enables Approximation of Time-varying Fokker-Planck Equation using Eigenvalues and Eigenvectors in Meta-state(spatial) and Time dimensions for NEXT Trade-day including Change-point.





Combined & Collapsed Matrices: ABC

Signal_3(meu)

	meu < 0.95	meu in [0.95,0.97]	meu > 0.97
Momentum(+)	0.14987	0.13310	0.19986
Momentum(0)	0.30065	0.33360	0.30041
Momentum(-)	0.00000	0.03316	0.03316

SIGNAL [ ] Combined & Collapsed Matrices: ABC

time

	60_m40	60_m20	60_m0
Momentum(+)	0.14987	0.13310	0.14987
Momentum(0)	0.30065	0.33360	0.30065
Momentum(-)	0.00000	0.03316	0.00340

The average probability under the curve is estimated using **vector cross-product** of dominant signal w.r.t. time and dominant signal w.r.t metastate. (right hand rule).

Integration bounaries are estimated by taking max and min of observed data points.

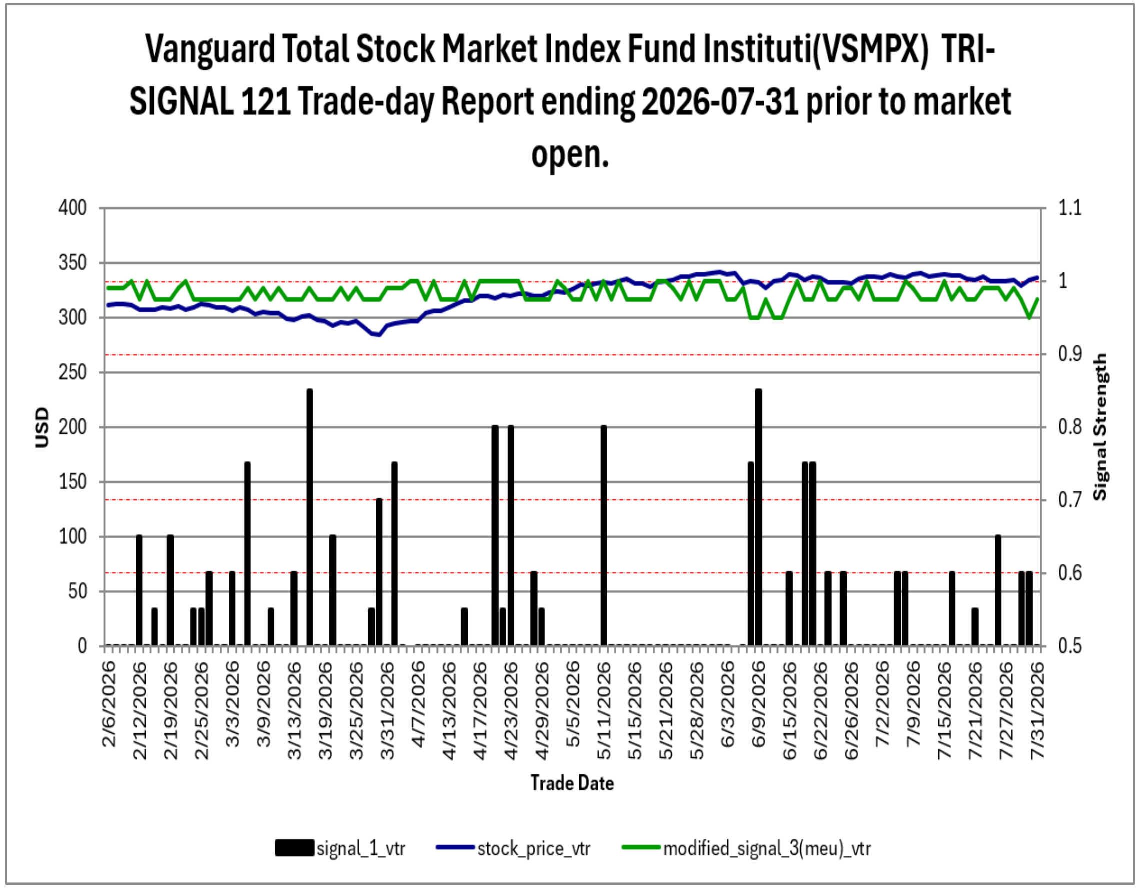
The average probability under the curve eigenvalue can also be calculated.

Sample AI-enabled Predictive Equity Analytics Report.

Vanguard Total Stock Market Index Fund Institution(VSMPX)

Ink blot changes from t-60 to t = 0

60 TD, t= 0	Momentum(+)	.	.	.00233835
	Momentum(0)	.	.00339329	.00156204
	Momentum(-)	.	.00479405	.00218549
60 TD, t= -20	Momentum(+)	.	.	.00257452
	Momentum(0)	.	.00339329	.00145183
	Momentum(-)	.	.00773251	.00253252
60 TD, t= -40	Momentum(+)	.	.	.00199161
	Momentum(0)	.	.	.00156605
	Momentum(-)	.	.	.00281421
60 TD, t= -60	Momentum(+)	.	.	.00238772
	Momentum(0)	.	.	.0014805
	Momentum(-)	.	.	.00333898



Sample AI-enabled Predictive Equity Analytics Report. (Continued) Download https://trisignalpea.com

Vanguard Total Stock Market Index Fund Institution(VSMPX)

List of (max 10) recent Retrospective change point detection (1

	Trade date	Offset	Close Price	Predicted Behavior	Next Trade-day Close P	True Up/Down Behavior
ABC, C High Signal EigenVector ChgPt(Time Dim.)	7/29/2026	-2	329.05	Up	334.4	Up
ABC,C EigenVector ChgPts(State space)	6/17/2026	-30	334.16	Up	337.97	Up

max	334.16
min	329.05

Retrospective Change-point Sum Conditional Probabilities Calculation 2

Direction	Forecast Count	Correct Count	Conditional Probability
Up	2	2	1
Down	0	0	1

The Henriksson and Mertons sufficient statistic for investment return value, defined as the sum of condition probabilities, exceeds 1.0 and approaches 2.0

Two retrospective change-points at t-30 and t-2 both "up."

List of (max 10) recent Prospective change point detection (90

	Trade date	Offset	Close Price	Predicted Behavior	Next Trade-day Close P	True_Behavior_Up_Dov	ABC Sigfig Signals	C Sigfig Signals
Next 1 to 3 Trade dates Prospective Change point detected.	7/30/2026	-1	334.4	UP	336.32	Up		
Next 1 to 3 Trade dates Prospective Change point detected.	7/29/2026	-2	329.05	UP	334.4	Up		
Next 1 to 3 Trade dates Prospective Change point detected.	7/24/2026	-5	333.07	UP	333.49	Up		
Next 1 to 3 Trade dates Prospective Change point detected.	7/16/2026	-11	338.46	UP	338.46	Up		
Next 1 to 3 Trade dates Prospective Change point detected.	7/8/2026	-17	336.38	UP	339.34	Up		
Next 1 to 3 Trade dates Prospective Change point detected.	7/7/2026	-18	337.61	UP	336.38	Down		
Next 1 to 3 Trade dates Prospective Change point detected.	6/25/2026	-25	332.34	UP	331.54	Down		
Next 1 to 3 Trade dates Prospective Change point detected.	6/23/2026	-27	332.27	UP	332.12	Down		
Next 1 to 3 Trade dates Prospective Change point detected.	6/18/2026	-29	337.97	UP	336.81	Down		
Next 1 to 3 Trade dates Prospective Change point detected.	6/17/2026	-30	334.16	UP	337.97	Up	ABC_Sigfig_Signal_Offset_20	C_Sigfig_Signal_Offs

No Significant prospective signals since t-30

max	338.46
min	329.05

Prospective Change-point Sum Conditional Probabilities Calculation 1.6

Accuracy: 0.6 Precision: 0.6 Sensitivity: 1 Specificity: 0

Direction	Forecast Count	Correct Count	Conditional Probability
Up	10	6	0.6

Instrument ty Mutual Fund

Most recent 46 trade dates.

Fokker-Pla ChgPt in Tir	Fokker-Planck Discrete ChgPt in Sta	As of Close Date Close Date	tate Behavi	Stock Price	As of Close Da State {0-26}	etrospectiv Action: Buy	Short run Local Path Dependent(S As of Close Daext Trade-da Predicted Buy/Sell/Nc	Short run Change in Drift(meu) Velocity Next Trade-day Predicted Action: Buy/Sell/No Action	Short run ignal_1 >= 0 ext Trade-d Predicted Buy/Sell/N	Long run Trade-day ext Trade-d Predicted verage Retu	Long run ,-60 Trade-d ext Trade-da Predicted verage Retu	Long run),-60 Trade-day Ro Next Trade-day Predicted Average Return
		7/31/2026	Up	336.32	20	Buy	0.9966, Sell	Drift(meu) chg. +5%, Strong Buy		1.0006	1.0009	.9995
		7/30/2026	Up	334.4	16	Buy	1.0057, Buy	Drift(meu) chg. -5%, Strong Sell	IFLECTION I	1.0003	1.0008	.9982
ABC, C High	Signal Eige	7/29/2026	Down	329.05	17	Sell	1.0081, Buy	Drift(meu) chg. -2%, Sell	Sell	1.0006	1.0009	.9994
		7/28/2026	Up	334.13	20	Buy	0.9966, Sell	Drift(meu) chg. +2%, Buy		1.0007	1.0011	.9994
		7/27/2026	Up	333.49	26	Buy	1.0014, Buy	Drift(meu) chg. -2%, Sell		1.0006	1.001	.9989
		7/24/2026	Horizontal	333.07	14	No Action	1.0035, Buy		No Action	1.0006	1.001	.9989
		7/23/2026	Down	333.07	23	Sell	999, No Action			1.001	1.0018	.9983
		7/22/2026	Up	337.51	20	Buy	0.9966, Sell	Drift(meu) chg. +2%, Buy		1.001	1.0018	.9983
		7/21/2026	Down	334.41	23	Sell	999, No Action			1.001	1.0018	.9983

Current Hedging Life / Annuity Products

Life Insurance and Annuity product Hedge Strategy assuming single distribution.

Key Takeaways

- **Indexed products** are often hedged statically, because cohorts can be identified and mostly independent and the index options typically well-defined by pure market parameters
- **Variable annuities** are typically hedged more dynamically, because the underlying liabilities are more dynamic, very long term, with open investments (funds/instead of indices) and require integrated inforce reporting

Slide: credit to Frank Zhang from Equity-Based Insurance Guarantees Conference

Risk neutral valuation and hedging under single distribution assumption.

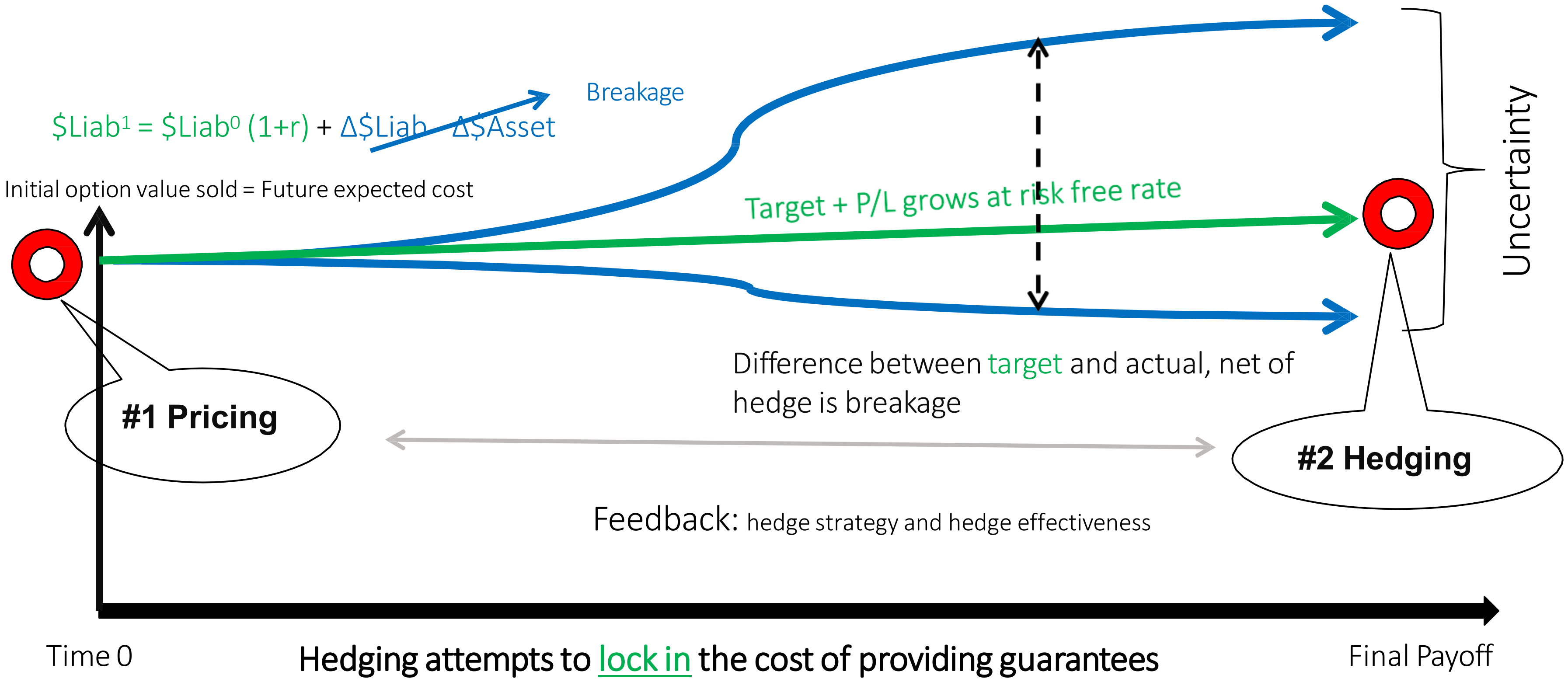
Key Takeaways

- **Risk neutral** valuation and hedging are easier to understand, because the liabilities grow at risk neutral (Martingale)
- **Risk management and hedging** are to reduce the cost of guarantees (initial option value) and to hedge to lock that in
- **If perfectly hedged**, cost of guarantee = cost of hedging. Otherwise add breakage.

- Two key components of (risk neutral) hedging

  #1 Reduce the expected cost of future guarantees – through product designs (the most effective)

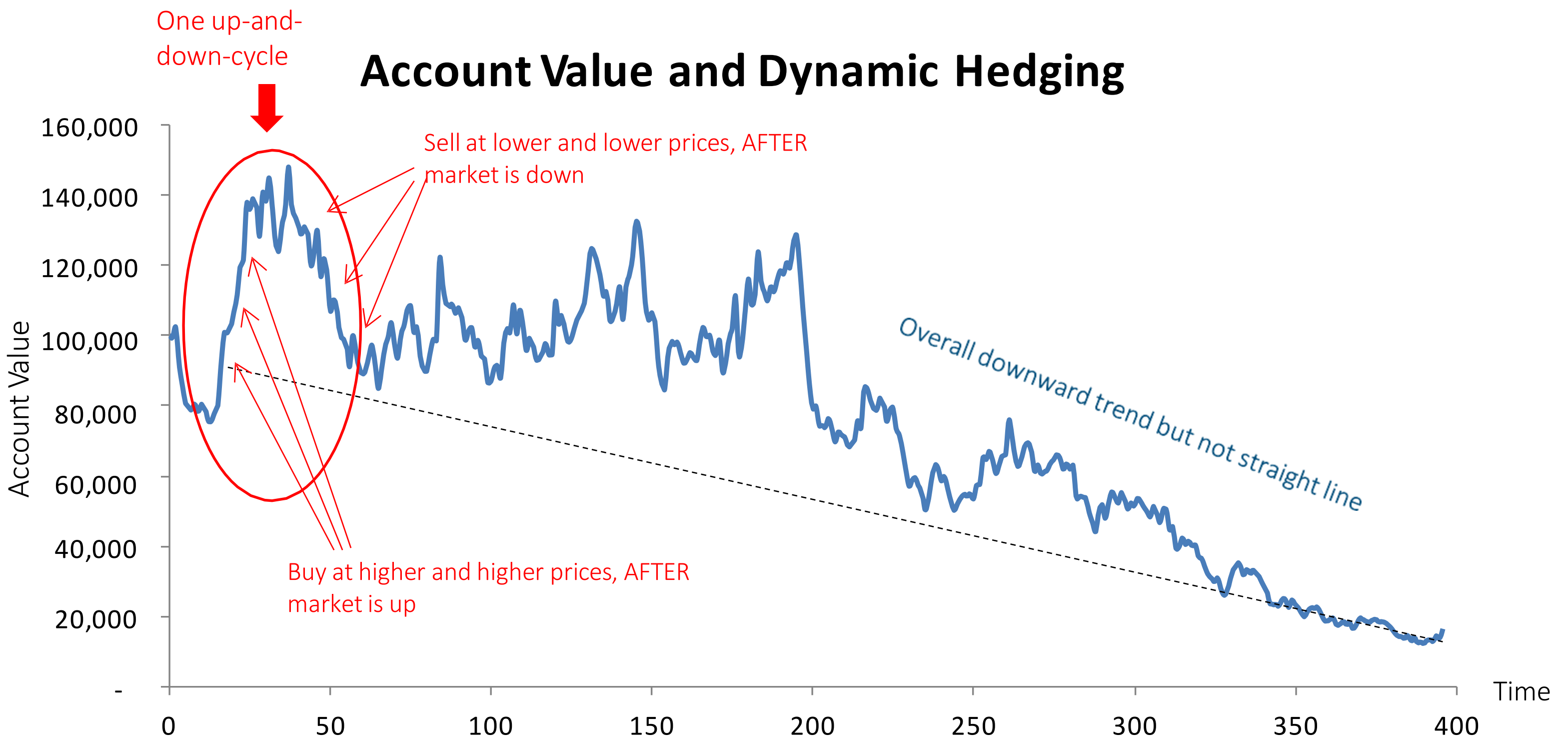
  #2 Reduce the uncertainty of cost of providing actual future guarantee claims – through hedging (sometimes the only tools)



Slide: credit to Frank Zhang from Equity-Based Insurance Guarantees Conference Chicago

Delta-1 dynamic hedging boils down to gamma risk under single distribution assumption.

- Higher realized volatility increases cost of dynamic hedging program
  - Hedge cost increases during periods when underlying funds moves sharply, resulting in elevated transaction costs and increased "buy-high-and-sell-low" round trip trades
  - Volatility drives cost of options
  - Dynamic hedging "premium" is spent gradually over the duration of the contract



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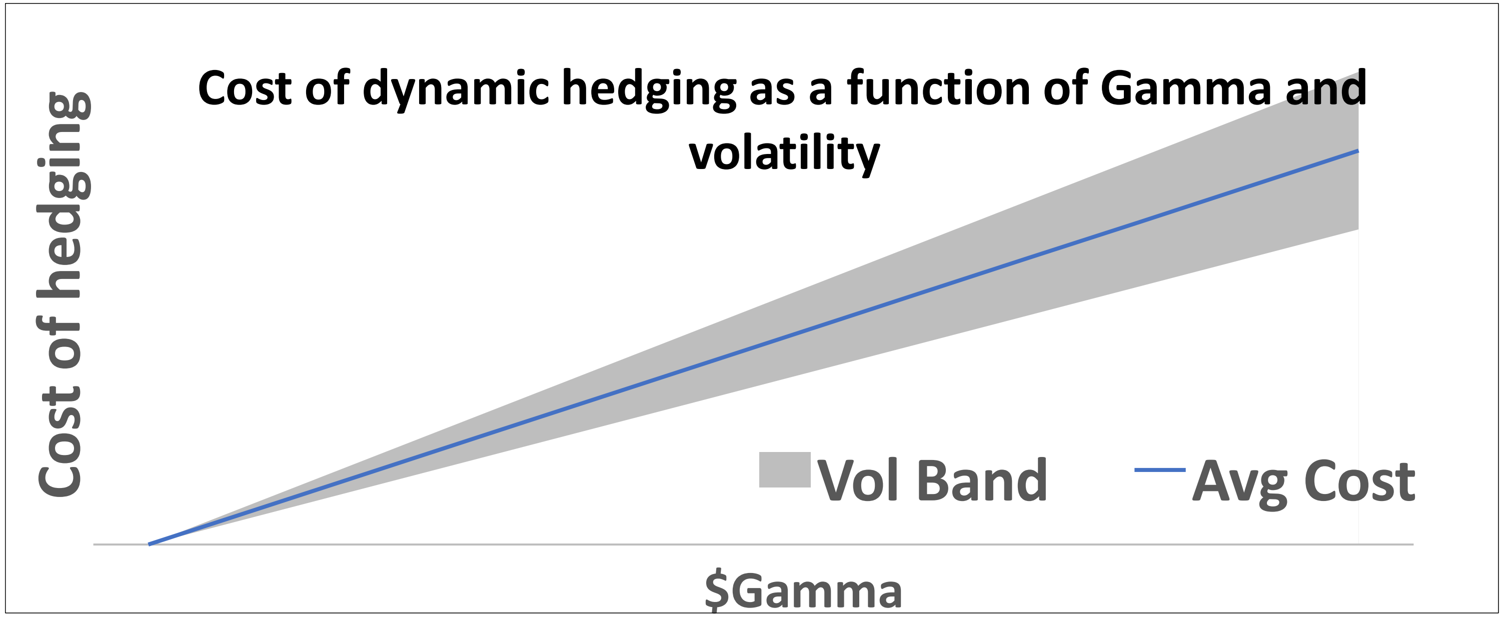
Delta-1 dynamic hedge cost in practice under single distribution assumption.

Key Takeaways

- Dynamic hedge cost is function of both Gamma and market volatility
  - Gamma is a function of derivative guarantee features and financial metrics of underlying business
  - Volatility is a function of capital market movements in the underlying funds



*Hedge Cost ~= ½ * Gamma * Vol^2*

Slide: credit to Frank Zhang from Equity-Based Insurance Guarantees Conference Chicago

AI-enabled Predictive Equity Analytics Hedging Life / Annuity Products

AI-enabled Predictive Equity Analytics Hedging relies on dynamic changepoint prediction/detection not on variance.

We have shown the single distribution underlying assumption is not necessarily valid and must first be tested to apply conventional variance based option framework.

(**New Hedging Technique**) Predictive Equity Analytics uses prospective/retrospective change-point prediction/detection each trade day and detects before hand change-point likely occurrence at which time proactive management is taken.

Is a form of just in time and right amount of hedging rather pre-purchase at beginning of period using options or derivatives.

Outline: How to use AI-enabled Predictive Equity Analytics to Hedge Equity Benefit Guarantee of 6% using Vanguard Total Stock Market Index Fund Institution(VSMPX)

Trade Date	Prospective Signal Retrospectiive Signal	Close Price	Action	Cumulative Return
				1.186626
8/7/2026	hold			
7/29/2026	Retrospective Changepoint	334.4	Buy	
7/16/2026	Signal_1 High	338.46	Sell	1.020872
6/25/2026	Signal_1 High	331.54	Buy	
6/17/2026	Retrospective Changepoint	337.97	Sell	1.153048
3/30/2026	Signal_1 High	293.11	Buy	
2/26/2026	Signal_1 High	309.41	Sell	1.0081
2/12/2026	Signal_1 High	306.93	Buy	

Summary

AI-enabled Predictive Equity Analytics with Dynamic State Space framework enables the user to avoid market downturns as well as capture upturns through prospective change point prediction and estimation of eigenvalue , eigenvector to determine retrospective changepoint behavior with highest accuracy.

Key Takeaways:

1. Equity time-series contain signals that can be sufficiently measured and mapped to a set of matrices: raw signal matrix ABC, filtered signal matrix C, and variance matrix AB.
2. Mixture of distributions for equity time series must be tested for rather than assuming single common underlying distribution.
3. Where no prospective changepoints and no retrospective changepoints then AI-enabled predictive equity analytics gracefully reverts to single distribution theory.
4. Using changepoint prediction/detection is called dynamic diversification and is the optimal portfolio framework when significant signals are present.
5. Ai-enabled predictive equity analytics enables just in time hedging which is the least cost hedging methodology.

Predictive reports that are easy to use and interpret include:

1.) retrospective changepoint sum conditional probabilities typically equals 2 (perfect forecast)

2.) prospective changepoint sum conditional probabilities exceeds 1 typically 1.5 or greater but mileage varies

3.) graphs, matrices, and

4.) dominant eigenvalue, eigenvector

AI-enabled Predictive Equity Analytics with Dynamic State Space has been operationalized.

- Five plus years of observation in US markets.
- Over 10,000+ equity symbols inclusive of ETFS, mutual funds and stocks.
- Accuracy exceeds 95%.
- 100% accurate avoiding market downturns.

To learn more, visit https://trisignalpea.com

APPENDIX

Theoretical Foundations: TRI-SIGNAL Predictive Equity Analytics with Dynamic State Space

What is at the heart of Predictive Equity Analytics with Dynamic State Space that "makes it work?"

Answer: "The core solution is a sufficiently accurate discrete approximation to the time-varying Fokker-Planck probability distribution transformation using eigenvalues and eigenvectors (i.e., linear algebra)."

Viewers are more familiar with Black-Scholes equation which is non-time varying Fokker-Planck equation.

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Drift as predictor

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Poisson outlier process and option pricing

- Merton, Robert (1976) "Option Pricing When Underlying Stock Returns are Discontinuous.", Journal of Financial Economics. Available at https://pages.stern.nyu.edu/~dbackus/Disasters/Merton%20jumps%20JFE%2076.pdf

Existence and uniqueness of discrete spectral solution (eigenvalue and eigenfunction) to time-varying Fokker-Planck probability transformation equation (Chapters 4 – 9 with chapters 1 -3 as foundation). [Spectral solution is theoretically valid (existence and uniqueness). Estimation of eigenvalues and eigenfunctions remained "a problem for applied research." From calculus, we approximate eigenfunctions with eigenvectors/lines for small domain neighborhoods.]

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  Chapters 1 – 4 of original manuscript available at: https://www.ma.imperial.ac.uk/~pavl/PavliotisBook.pdf

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Point-wise mapping of equity time-series into set of 3x3 matrices analysis discussion.

- Any point-wise mapping must be 100% inclusive and verifiably complete in terms of sample size and preservation of data point values.
- Any deterministic or random mapping method is unlikely to yield consistently informative set of matrices over a sequence of contiguous trade dates.
- Necessary and sufficient condition for 3x3 mapping existence is generation of signal vector for each data point such that the tagged signal vectors when mapped into 3x3 set of matrices results in non-random set of matrices. Said another way, the set of matrices has structure (eigen analysis) and is informative with respect to prospective and retrospective change-point prediction/detection. Proof by contradiction.